

Efficient Multiagent Planning via Shared Action Suggestions

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ABSTRACT

Decentralized partially observable Markov decision processes with communication (Dec-POMDP-Com) provide a framework for multiagent decision making under uncertainty, but the NEXP-complete complexity renders solutions intractable in general. While sharing actions and observations can reduce the complexity to PSPACE-complete, we propose an approach that bridges POMDPs and Dec-POMDPs by communicating only suggested joint actions, eliminating the need to share observations while maintaining performance comparable to fully centralized planning and execution. Our algorithm estimates joint beliefs using shared actions to prune infeasible beliefs. Each agent maintains possible belief sets for other agents, pruning them based on suggested actions to form an estimated joint belief usable with any centralized policy. This approach requires solving a POMDP for each agent, reducing computational complexity while preserving performance. We demonstrate its effectiveness on several Dec-POMDP benchmarks showing performance comparable to centralized methods when shared actions enable effective belief pruning. This action-based communication framework offers a natural avenue for integrating human-agent cooperation, opening new directions for scalable multiagent planning under uncertainty, with applications in both autonomous systems and human-agent teams.

Repository: https://github.com/dylan-asmar/estimated_joint_belief

KEYWORDS

Dec-POMDP, Dec-POMDP-Com, Multiagent Planning, Sequential Decision Making

1 INTRODUCTION

From scientific research and complex engineering projects to emergency response teams and military operations, effective coordination between individuals is vital for success. The ability of humans to work together, communicate intuitively, and adapt to changing conditions has inspired researchers to explore cooperation in autonomous systems [1]. However, achieving the same seamless collaboration in autonomous teams remains a significant challenge.

In the context of multiagent decision making under uncertainty, where agents must act without complete knowledge of the state of their environment and outcomes of actions are stochastic, one widely used model is the decentralized partially observable Markov decision process (Dec-POMDP) [10]. Agents must reason not only about their environment but also about the possible actions and beliefs of other agents without directly communicating. Dec-POMDPs are powerful, but notoriously hard to solve—making them impractical for many real-world problems [33].

When agents are allowed to communicate, the computational burden can be reduced under certain assumptions [20, 36]. However, sharing the required information can become impractical in terms

of complexity and the cost of communication. In addition, when the communication is not lossless and free, the complexity of a Dec-POMDP with communication (Dec-POMDP-Com) remains NEXP-complete [21].

The challenges in multiagent coordination become even more pronounced when we consider the growing field of human-agent teaming. As autonomous systems become more capable, the idea of humans and machines working together to solve problems becomes increasingly relevant [14, 26]. The hope is that combining human intuition and adaptability with the computational power and efficiency of machines will create teams that outperform either humans or machines working alone. However, realizing this potential requires addressing not only the complexities of multiagent collaboration but also the unique challenges of human-machine communication and shared understanding [13, 22].

Humans and autonomous agents often operate using different models of the world and decision-making processes, and finding a way for them to communicate effectively is crucial for collaboration [40]. In many cases, humans communicate by simply suggesting actions—“let’s move there” or “take that route”—without needing to share all the details of their observations or beliefs. For instance, when a friend suggests, “We should eat at restaurant X,” they are not just proposing an action, but implicitly communicating several beliefs: that the restaurant is open, that it fits the group’s dietary needs and preferences, that it is within an acceptable price range, and possibly that it is not too crowded at the moment. The suggestion encapsulates a complex set of observations and reasoning in a simple action proposal.

This type of action-based communication is natural for humans but has not been fully explored as a method for enabling collaboration in autonomous systems or human-agent teams. In this paper, we propose an approach that narrows the focus of communication to suggested joint actions. Instead of sharing raw observations or beliefs, agents communicate their recommended actions at each step. This approach mirrors how humans often collaborate in complex tasks—by using action suggestions to convey important information.

Our proposed method estimates joint beliefs by maintaining sets of reachable beliefs and inferring the beliefs of the other agents. The key insight is that an action suggestion implies the agent’s belief is within a particular subspace of the belief space. We can use that information to prune infeasible beliefs from the belief set. The agent can then more accurately infer the other agents’ beliefs, enabling the construction of an estimated joint belief that can be used with a policy assuming centralized execution. This method requires solving n multiagent POMDPs (MPOMDPs) for an n agent problem, online computation of belief updates for all of the beliefs in the belief set, and a joint policy using the centralized assumptions (solving another MPOMDP). We evaluate this approach on several

standard Dec-POMDP benchmarks and more complex variations of the standard problems. The results demonstrate our approach performs similarly to a fully centralized method when the shared action information provides effective belief pruning.

2 RELATED WORK

This work builds upon key areas in decentralized decision making, including communication in Dec-POMDPs, sufficient statistics for planning, and action-based coordination methods.

Communication in Dec-POMDPs. The introduction of communication to Dec-POMDPs has been explored as a way to reduce the computational burden and improve coordination among agents. Pynadath and Tambe [36] examined how communication strategies could improve multi-agent teamwork, focusing on balancing the benefits of shared information with the practical constraints of decentralized environments. Goldman and Zilberstein [20] further investigated the optimization of information exchange in these models, showing how selective communication can enhance decision-making.

Sufficient Statistics. The concept of sufficient statistics has played an important role in simplifying the planning process for Dec-POMDPs. Oliehoek [32] introduced the idea of a probability distribution over joint action-observation histories as a sufficient plan-time statistic for the past joint policy. Dibangoye et al. [15] recast Dec-POMDPs as a continuous state MDP using occupancy states, allowing the application of POMDP techniques. While we do not directly adopt the occupancy MDP framework, our method shares the goal of compactly representing the system’s information state for more efficient planning and execution.

One-Sided Information Sharing. One-sided information sharing has been studied as a method for reducing the complexity of Dec-POMDPs by allowing one agent to have access to both its own and the other agent’s observations. Xie et al. [42] demonstrated that in settings with one-sided information sharing, where one agent has full access to both its own and the other agent’s observations, the informed agent can act as a central planner, coordinating decisions optimally.

Action-Based Coordination. This work is also related to research on action-based coordination in multi-agent systems. Previous work has explored the use of suggested actions as a means of communication between agents, treating these suggestions as observations of the environment [8]. However, that work assumed that suggested actions were conditioned on the true state of the environment, which becomes less reliable when agents make suggestions based on their beliefs about the state rather than the state itself.

A related example of action-based coordination can be found in aircraft collision avoidance systems like TCAS and ACAS X [7]. These systems use action advisories to restrict the actions of other aircraft, effectively coordinating decisions without direct observation or state sharing. The systems issues advisories like “do not descend,” which restrict the set of available actions for other aircraft. ACAS X performs this by adding online costs to the incompatible actions to help ensure cooperative behavior [6].

The Proposed Approach in Context. This work builds on the idea that communication can help reduce computational complexity by using communication of suggested joint actions. These suggested actions are used to construct a distribution over the beliefs of the other agents, providing distribution over sufficient statistics for the histories. Unlike previous work using action suggestions, this method allows both agents to interact with the environment where they all have partial observability. It leverages the ideas of one-sided information sharing where an agent can select optimal joint actions if it knows the histories of the other agents, but relaxes the assumption of full observation access. By relying on joint action suggestions, this approach reduces the need for full communication while maintaining coordination efficiency and it also provides a natural framework for extending this approach to human-agent teaming where action suggestions are an intuitive mode of communication.

3 BACKGROUND

A partially observable Markov decision process (POMDP) is a mathematical framework to model sequential decision making problems under uncertainty [38]. A POMDP is represented as a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, R, \gamma)$, where \mathcal{S} is a set of states, \mathcal{A} is a set of actions, and \mathcal{O} is a set of observations. At each time step, an agent in state $s \in \mathcal{S}$ chooses an action $a \in \mathcal{A}$, transitions to s' based on $T(s, a, s') = P(s' | s, a)$, and receives an observation $o \in \mathcal{O}$ based on $O(s', a, o) = P(o | s', a)$.

The agent receives a reward $R(s, a) \in \mathbb{R}$, with discount factor $\gamma \in [0, 1)$ for infinite horizons. The goal is to maximize the total expected reward $\mathbb{E} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \right]$, where s_t and a_t are the state and action at time t . One method to solve a POMDP is to infer a belief distribution $b \in \mathcal{B}$ over \mathcal{S} and then solve for a policy π that maps the belief to an action where \mathcal{B} is the set of beliefs over \mathcal{S} [27]. Executing with this type of policy requires maintaining b through updates after each time step.

A Decentralized POMDP (Dec-POMDP) extends the POMDP framework to multiple cooperative agents. It can be represented as a tuple $(\mathcal{I}, \mathcal{S}, \{\mathcal{A}^i\}, \{\mathcal{O}^i\}, T, O, R, \gamma)$, where \mathcal{I} is the set of agents, and each agent $i \in \mathcal{I}$ selects a local action $a^i \in \mathcal{A}^i$ and receives a local observations $o^i \in \mathcal{O}^i$. In this paper, we use superscripts to represent the agent index and bold variables to represent the joint collection across all agents, e.g., $\mathbf{a} = (a^1, \dots, a^{|\mathcal{I}|})$. The true state of the system $s \in \mathcal{S}$ is shared by all agents, while the reward, transition, and observation functions are defined over joint actions and observations (i.e., $R(s, \mathbf{a})$, $T(s, \mathbf{a}, s')$, and $O(s', \mathbf{a}, \mathbf{o})$) [27, 33].

In many scenarios, agents often have the ability to communicate. A Dec-POMDP with communication (Dec-POMDP-Com) further extends the Dec-POMDP framework by allowing communication between agents. The Dec-POMDP tuple remains the same with the addition of $\{\Sigma^i\}$ and C_{Σ} where Σ^i is the alphabet of possible messages that agent i can send and C_{Σ} is the communication cost function [33, 36].

In both a Dec-POMDP and a Dec-POMDP-Com, agents must make decisions based on their individual action-observation histories (and messages received in a Dec-POMDP-Com), as they do not have access to the full state or the observations of other agents. The goal is to find a joint policy $\boldsymbol{\pi} = (\pi^1, \dots, \pi^{|\mathcal{I}|})$ that maximizes

the expected discounted sum of the shared rewards. Solving a finite horizon Dec-POMDP or a Dec-POMDP-Com is NEXP-complete [10, 33]. If the agents can communicate their actions and observations perfectly and without cost, then the agent’s can maintain a collective belief state and this model is called a multiagent POMDP (MPOMDP). MPOMDPs can be solved using the same approaches used to solve POMDPs [33, 36].

In our approach, we use MPOMDP policies instead of solving the Dec-POMDP directly. Policies for POMDPs and MPOMDPs can be generated offline or computed online during execution. In this work, we integrate our method with policies generated offline and leave the application to online solvers for future work. In particular, we use SARSOP [28] to generate the policies and represent the policy as a set of alpha vectors, but our approach is not limited to SARSOP or alpha vectors and can be applied to policies generated by other methods.

4 PROBLEM FORMULATION AND NOTATION

The problem we are focusing on is a collaborative sequential decision-making problem under uncertainty and fits within the Dec-POMDP-Com framework. We perform our experiments assuming infinite horizon problems, but the methods could apply to finite horizon problems as well. In this work, we assume discrete state, action, and observation spaces.

The alphabet of messages for each agent is equal to the action space for that agent $\Sigma^i = \mathcal{A}^i$ and the messages are sent without cost. We further assume that each agent sends its message after receiving its local observation and before performing an action. We do not model any message noise or loss and assume all messages are received. We denote the message from agent i to agent j at time t as $\sigma_t^{i,j} \in \Sigma^i$.

As mentioned in section 3, a single superscript is the agent index and bold variables are the joint collection across all agents, e.g., $\mathbf{a} = (a^1, \dots, a^I)$. Each agent maintains a belief over the state space, updated based on local observations. The belief of agent i at time t will be designated as $b_t^i \in \mathcal{B}^i$ where \mathcal{B}^i is the belief space of agent i . We use a tilde instead of a bold symbol to indicate a joint belief \tilde{b} since the joint belief is not a collection.

We also assume that each agent has access to a surrogate policy for other agents. The surrogate policy $\hat{\pi}^{i,j}$ is the policy agent i assumes agent j is operating with. In environments like our experiments where we conduct centralized planning offline, the surrogate policy equals the true policy $\hat{\pi}^{i,j} = \pi^j$.

In this problem setting, agents will be maintaining estimates with respect to the other agents (e.g. estimates of other agents’ beliefs). Any estimation will be marked with a *hat* symbol. A superscript of two indices i, j on an estimation refers to the item belonging to agent i , about agent j . For example, $\hat{b}_{t_k}^{i,j}$ represents the k^{th} estimated belief agent i has for agent j at time t , and the set of estimated beliefs agent i has for agent j will be designated as $\hat{\mathcal{B}}^{i,j}$.

In a slight abuse of notation, we use subscripts to indicate the time step (b_t), counting of the number of variables of a collection (subscript to the time step, b_{t_k}), and for indexing actions and observations (a_t, o_m). When a subscript is used on an action or observation, we are referencing the index of that action within the action space, e.g. $a_t^i \in \mathcal{A}^i$ is the t^{th} action in \mathcal{A}^i .

5 USING ACTION SUGGESTIONS

There are several ways agents can use suggested actions. The simplest option is to ignore the messages and choose actions as if there was no communication, which is equivalent to a Dec-POMDP. Alternatively, agents could designate a leader at each time step and follow the leader’s suggested actions, which is sufficient in some environments where one agent’s observations provide enough information, as in the Broadcast Channel problem (section 6.3). Another approach is hierarchical action selection, where agents select actions and communicate following a specific communication order. In this scheme, each agent can select an action with knowledge of the previous messages received for that time step. The order of communication becomes important as agents earlier in the process have to make decisions with less information. This approach is similar to other prioritization schemes [17].

In our approach, we use suggested actions to infer beliefs. In a cooperative scenario, we assume agents act optimally to maximize shared rewards. Therefore, we assume the suggested action is the one that maximizes the expected sum of discounted rewards based on the agent’s belief of the environment. Referencing back to the restaurant example from the introduction, we can infer aspects of the friend’s belief from their action suggestion by assuming they are acting optimally and want to maximize the happiness of the group. For instance, if a friend suggests a restaurant, we can infer they believe it is open and suitable for the group’s preferences. Each action suggestion thus contains information related to the suggester’s belief of the environment, which we can use to infer their belief.

5.1 Inferring the Belief Subspace

We can use the suggested action and the fact that the suggested action is the optimal action from the suggester’s perspective to infer the possible beliefs the agent could have. For example, if agent i receives a suggested action a_s from agent j using policy π^j , then we know $b^j \in \mathcal{B}_{a_s}^j$ where $\mathcal{B}_{a_s}^j = \{b \mid \pi^j(b) = a_s, \forall b \in \mathcal{B}^j\}$.

In an alpha vector policy, this would be the subspace of beliefs that are dominated by vectors associated with the suggested action. With a set of alpha vectors Γ representing the policy and a suggested action a_s

$$\mathcal{B}_{a_s}^j = \{\mathbf{b} \mid (\alpha_i - \alpha_j) \cdot \mathbf{b} \geq 0, \quad \forall \alpha_i \in \Gamma_{a_s}, \forall \alpha_j \in \Gamma\} \quad (1)$$

where $\Gamma_{a_s} \subseteq \Gamma$ is the set of alpha vectors corresponding to action a_s .

Figure 1 provides a graphical example of this subspace where we have a simple environment with two states and the x axis represents the probability of being in the first state. The notional alpha vector policy consists of six alpha vectors and the region indicated as $\mathcal{B}_{a_1}^j$ is the subspace dominated by a_1 alpha vectors. Therefore, we know that if agent j is acting optimally using this policy, then $b^j(s=1) \in [0, 1] \cup [0.7, 0.8]$.

5.2 Inferring the Belief

5.2.1 Pruning Beliefs. At each time step, agents update their beliefs based on individual observations and actions performed. From agent i ’s perspective, there are $|O^j| \prod_{i \neq j} |\mathcal{A}^k|$ possible beliefs reachable from $\hat{b}_t^{i,j}$ for agent j . The size of this set grows exponentially in time,

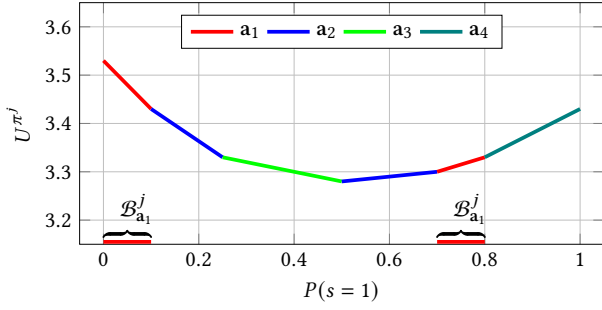


Figure 1: Example of the dominated belief subspace in an alpha vector policy for action a_1 .

reaching $(|O^j| \prod_{i \neq j} |\mathcal{A}^i|)^\ell$ after ℓ time steps. This exponential growth is one of the primary factors in the NEXP complexity of solving Dec-POMDPs.

To help manage this growth, we can prune infeasible beliefs using the suggested actions. We can rigorously define the belief subspace in which the suggester’s belief must lie (eq. (1)) and this subspace is an infinite set of beliefs. While we cannot easily construct the subspace, we can test if a belief is within this subspace by evaluating the policy at that belief.

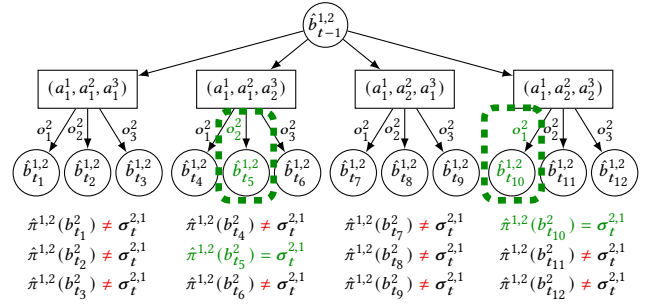
Without loss of generality, we will discuss this process from the perspective of agent i maintaining a belief estimate for agent j . We start with an initial belief set $\hat{\mathcal{B}}_0^{i,j} = b_0^j$, where in our approach, we assume all agents begin with the same initial belief. After performing an action and receiving a local observation, we expand the beliefs considering all possible actions and observations, resulting in $|\hat{\mathcal{B}}_t^{i,j}| = |\hat{\mathcal{B}}_{t-1}^{i,j}| |O^j| \prod_{j \neq i} |\mathcal{A}^j|$ at time t . We then evaluate each belief with the surrogate policy for agent j and prune the beliefs where the optimal action does not match the received message

$$\hat{\mathcal{B}}_t^{i,j} \leftarrow \{b \in \hat{\mathcal{B}}_{t-1}^{i,j} \mid \hat{\pi}^{i,j}(b) = \sigma^{j,i}\}. \quad (2)$$

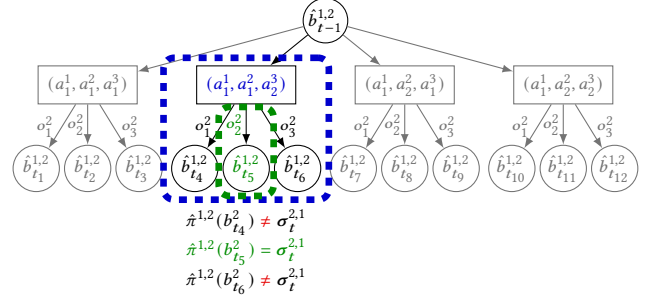
If we know the actions performed at the last time step, we only need to consider observations for a single joint action, increasing our estimated belief set by a factor of $|O^j|$. This knowledge significantly reduces the size of the reachable belief set.

Figure 2 illustrates this pruning process. In this example with three agents ($n = 3$), each agent has two possible actions ($|\mathcal{A}^i| = 2$) and can receive one of three observations ($|O^i| = 3$). The figure shows agent 1 updating a single estimated belief for agent 2. In fig. 2a, where agent 1 only knows its own action (a_1^1), all 12 possible beliefs must be checked. After pruning based on alignment with the received message, only $\hat{b}_{t_5}^{1,2}$ and $\hat{b}_{t_{10}}^{1,2}$ remain. Figure 2b demonstrates that if the joint action at the last time step were known, only 3 beliefs would need to be checked, and after pruning, only a single belief ($\hat{b}_{t_5}^{1,2}$) would remain.

5.2.2 Similar Beliefs. After pruning the infeasible beliefs, we can further reduce our set by removing beliefs that are sufficiently close to other beliefs in the set. Zhang et al. [43] showed that for any two beliefs b and b' , if $\|b - b'\|_1 \leq \delta$, then $|P(o \mid b, a) - P(o \mid b', a)| \leq \delta$. Additionally, Hsu et al. [25] proved that the value function of POMDPs satisfies the Lipschitz condition, i.e., $|V(b) - V(b')| \leq$



(a) Pruning all reachable beliefs by removing beliefs that don’t align with the received message $\sigma_t^{2,1}$.



(b) Example of the reduction of the reachable belief size if the joint action was known. Of the remaining beliefs, only $\hat{b}_{t_5}^{1,2}$ is equal to the received message (i.e. the received action suggestion).

Figure 2: Example of pruning reachable beliefs. This example has $n = 3$, $|\mathcal{A}^i| = 2$, and $|O^i| = 3$. The process is from agent 1’s perspective, expanding a single belief estimate for agent 2.

$\frac{\|R\|_\infty}{1-\delta} \delta$ if $\|b - b'\|_1 \leq \delta$ and Wu et al. [41] used this bound to combine beliefs in their proposed POMDP algorithm. Building on this previous work, we can further reduce the size of our reachable belief set by removing beliefs within the same δ -ball for some parameter δ_t .

5.3 Joint Belief Estimation

5.3.1 Combining Beliefs. After inferring beliefs of other agents, we must combine these with the receiving agent’s own belief to estimate a joint belief. Various methods exist for combining probability distributions [19]. One straightforward approach is to form a mixture distribution. Agent i ’s estimated joint belief \hat{b}^i would be

$$\hat{b}^i = w^i b^i + \sum_{j \neq i} w^j \hat{b}^{i,j} \quad (3)$$

where $\sum_{j=1}^n w^j = 1$. While intuitive, this method requires assigning and justifying potentially unequal weights.

An alternative method is conflation [24]

$$\hat{b}^i(s) = \frac{b^i(s) \prod_{j \neq i} \hat{b}^{i,j}(s)}{\sum_{s' \in \mathcal{S}} b^i(s') \prod_{j \neq i} \hat{b}^{i,j}(s')} \quad (4)$$

Unlike many methods (e.g., weighted averages), conflation is not idempotent (i.e., $T(P, \dots, P) = P$), which can be beneficial when consolidating results from independent observations. As noted by

Hill [24], conflation does not require ad hoc weights, allows for flexible representation of uncertainty through potential increases or decreases in mean and variance, automatically prioritizes more accurate beliefs by giving more weight to distributions with smaller standard deviations, and minimizes the loss of Shannon information when consolidating multiple distributions into a single one.

5.3.2 Infer Observation. When inferring other agents’ beliefs, we have access to the inferred action-observation histories that would lead to these estimated beliefs. Rather than using the beliefs directly, we can leverage these inferred actions and observations to update an estimated joint belief. This process would double the number of beliefs we have to maintain in memory and double the number of belief updates we would have to perform; however, it would avoid any issues with combining distributions and allow for a more thorough consideration of non-independent observations.

Using the estimates of the observations and actions, we can update our estimated joint belief

$$\hat{b}_t^i(s') \propto O(\hat{o} \mid \hat{a}, s') \sum_s T(s' \mid s, \hat{a}) \hat{b}_{t-}^i(s) \quad (5)$$

where $\hat{o} = (\hat{o}^1, \dots, \hat{o}^i, \dots, \hat{o}^n)$, $\hat{a} = (\hat{a}^1, \dots, \hat{a}^i, \dots, \hat{a}^n)$, and \hat{b}_{t-}^i is the estimation from the previous time step.

5.3.3 Belief and Action Selection. Using the suggested joint actions to prune the reachable beliefs and removing similar beliefs is effective in reducing the size of our estimated belief set. However, the belief subspace dominated by the suggested action can be composed of disjoint subsets, and pruning does not guarantee the reduction to a single belief. These two cases are shown in figs. 1 and 2a.

To form our set of estimated joint beliefs, we combine all possible estimated beliefs of the other agents. The number of possible estimated joint beliefs is $\prod_{j \neq i} |\hat{\mathcal{B}}^{i,j}|$. In practice, when the information implied by an action results in a small belief subspace, we often do not have many beliefs to consider. We demonstrate this in our experiments by sharing the alpha vector index instead of the action, thus sharing a single subspace region that is dominated by the optimal action. However, in cases where an action does not imply much information, the pruning is less effective, and we must employ selection strategies.

We do not combine estimations of joint beliefs (e.g., through centroids or weighted averages) because they may represent different beliefs resulting from different observation sequences. Instead, we maintain counts for each unique belief. When pruning similar beliefs we increment the count for the retained belief. These counts serve as weights in our selection process, indicating how many paths led to each belief.

When selecting a single belief from multiple candidates, we choose the one with the highest count, as it represents the most frequently reached belief state through different paths. In the case of ties, we use random selection to avoid bias. We then use this selected estimated joint belief to choose an action using a policy based on the assumption of shared observations and actions (a centralized joint policy).

This approach of maintaining belief counts and selecting based on weights provides a good balance between computational efficiency and decision quality in our experiments. However, the

effectiveness can vary depending on the specific characteristics of the problem being solved. There are potential areas for future improvements, such as implementing history-based selection for more nuanced belief choice, developing more sophisticated action selection strategies like regret minimization across all estimated joint beliefs [9], and further research into optimal belief and action selection strategies for various problem scenarios. These enhancements could potentially improve performance in scenarios where our current method is less effective, but we leave their exploration for future work.

5.4 Multiagent Control via Action Suggestions (MCAS) Algorithm

Our approach begins by solving $n + 1$ MPOMDPs. For each agent $i \in 1, \dots, n$, we solve an MPOMDP where agent i receives individual observations (observation space O^i) but has control over all agents (action space $\mathcal{A}^1 \times \mathcal{A}^2 \times \dots \times \mathcal{A}^n$). This results in policies π^1, \dots, π^n . We also require a policy $\tilde{\pi}$ that assumes joint observations and uses a joint belief, which can be generated by any suitable solver (online or offline).

The MCAS algorithm (algorithm 1) operates from the perspective of agent 1, arbitrarily designated as the coordinating agent. This approach builds upon leader-based coordination but differs by integrating information from all agents. Unlike hierarchical action selection, it does not rely on a fixed communication order, instead treating all agents’ suggestions equally to infer a comprehensive joint belief. The coordinating agent receives action suggestions from others, estimates a joint belief, and suggests a final joint action based on the centralized policy, which all agents then follow.

The COMBINEBELIEFS function (line 20) can be implemented using various methods such as weighted averaging or conflation (section 5.3.1). If maintaining estimated joint beliefs from inferred observations, the UPDATEESTBELIEFS function (line 31) would need to return associated observations, and the belief combination process would involve updates for all possible observation combinations, potentially improving the accuracy of the joint belief estimate at the cost of increased computational complexity.

Pruning based on the suggested action is effective in practice; however, the number of reachable beliefs can still grow exponentially in the worst case. The REDUCETOMAXLIMIT function (line 7) limits the size of the belief set to \bar{B}_{\max} . Our implementation computes the $\mathcal{L}1$ norm between all belief pairs, sorts these distances, and iteratively removes the lower-weighted belief of the closest pair, adding its weight to the remaining belief, until reaching \bar{B}_{\max} .

6 EXPERIMENTS

To evaluate our approach, we conducted experiments on various Dec-POMDP benchmarks. Initial tests using a leader-based approach, where one agent controls the group based on its individual observations, revealed that in some problems, individual observations contained sufficient information, limiting the benefit of integrating observations from other agents. Consequently, we introduced modifications to standard problems to emphasize coordination and demonstrate the value of integrating different beliefs.

Algorithm 1: Multiagent Control via Action Suggestions

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Given:  $n$  /* Number of agents */
 $\mathcal{P}^1, \dots, \mathcal{P}^n$  /* Agents' MPOMDPs */
 $\pi^1, \dots, \pi^n$  /* Agents' policies */
 $\hat{\mathcal{P}}, \hat{\pi}$  /* Joint MPOMDP and policy */
 $\delta_{\text{joint}}, \delta_{\text{single}}$  /* Similarity thresholds */
 $\bar{B}_{\text{max}}$  /* Maximum number of estimated beliefs */

1 Initialize belief  $b^1$  for agent 1
2 Initialize surrogate belief sets  $(\hat{\mathcal{B}}^{1,j}, w^{1,j}) = \{(b_0^j, 1.0)\}$  for  $j = 2, \dots, n$ 
3 while not done do
4   Receive messages  $\sigma^{j,1}$  from agents  $j = 2, \dots, n$ 
5   for  $j \leftarrow 2$  to  $n$  do
6      $\hat{\mathcal{B}}^{1,j} \leftarrow \text{PRUNEBELIEFS}(\pi^j, \hat{\mathcal{B}}^{1,j}, \sigma^{j,1})$ 
7      $\hat{\mathcal{B}}^{1,j} \leftarrow \text{REDUCETOMAXLIMIT}(\hat{\mathcal{B}}^{1,j}, \bar{B}_{\text{max}})$ 
8    $\hat{b} \leftarrow \text{SELECTJOINTBELIEF}(\{(\hat{\mathcal{B}}^{1,j}, w^{1,j})\}_{j=2}^n, b^1, \delta_{\text{joint}})$ 
9    $\hat{a} \leftarrow \hat{\pi}(\hat{b})$ 
10  Broadcast  $\hat{a}$  to all agents
11  Execute  $\hat{a}[1]$  and observe  $o^1$  /* Agent 1's action */
12   $b^1 \leftarrow \text{UPDATE}(\mathcal{P}^1, b^1, \hat{a}, o^1)$ 
13  for  $j \leftarrow 2$  to  $n$  do
14     $\hat{\mathcal{B}}^{1,j}, w^{1,j} \leftarrow \text{UPDATEESTBELIEFS}(j, \mathcal{P}^j, \hat{\mathcal{B}}^{1,j}, w^{1,j}, \hat{a}, \delta_{\text{single}})$ 

15 Function  $\text{PRUNEBELIEFS}(\pi, \hat{\mathcal{B}}, \sigma)$ 
16   return  $\{b \in \hat{\mathcal{B}} \mid \pi(b) = \sigma\}$ 

17 Function  $\text{SELECTJOINTBELIEF}(\{(\hat{\mathcal{B}}^j, w^j)\}_{j=2}^n, b^1, \delta_{\text{joint}})$ 
18    $\mathcal{B}_{\text{combined}} \leftarrow \emptyset, w_{\text{combined}} \leftarrow \emptyset$ 
19   for  $(\hat{b}^2, \dots, \hat{b}^n) \in \hat{\mathcal{B}}^2 \times \dots \times \hat{\mathcal{B}}^n$  do
20      $b^c \leftarrow \text{COMBINEBELIEFS}(b^1, \hat{b}^2, \dots, \hat{b}^n)$ 
21      $w^c \leftarrow \prod_{j=2}^n w^j[\text{index}(\hat{b}^j)]$ 
22     if  $\forall b' \in \mathcal{B}_{\text{combined}} : \|b^c - b'\|_1 \geq \delta_{\text{joint}}$  then
23        $\mathcal{B}_{\text{combined}} \leftarrow \mathcal{B}_{\text{combined}} \cup \{b^c\}$ 
24        $w_{\text{combined}} \leftarrow w_{\text{combined}} \cup \{w^c\}$ 
25     else
26        $k \leftarrow \text{argmin}_{b' \in \mathcal{B}_{\text{combined}}} \|b^c - b'\|_1$ 
27        $w_{\text{combined}}[k] \leftarrow w_{\text{combined}}[k] + w^c$ 
28    $w_{\text{normalized}} \leftarrow w_{\text{combined}} / \|w_{\text{combined}}\|_1$ 
29    $k \leftarrow \text{argmax}_i w_{\text{normalized}}[i]$ 
30   return  $\mathcal{B}_{\text{combined}}[k]$ 

31 Function  $\text{UPDATEESTBELIEFS}(j, \mathcal{P}, \hat{\mathcal{B}}, w, a, \delta_{\text{single}})$ 
32    $\hat{\mathcal{B}}_{\text{new}} \leftarrow \emptyset, w_{\text{new}} \leftarrow \emptyset$ 
33   for  $i \leftarrow 1$  to  $|\hat{\mathcal{B}}|$  do
34     for  $o \in \mathcal{O}^j$  do
35        $b' \leftarrow \text{UPDATE}(\mathcal{P}, \hat{\mathcal{B}}[i], a, o)$ 
36        $w' \leftarrow w[i] \cdot P(o \mid \hat{\mathcal{B}}[i], a)$ 
37       if  $\forall b'' \in \hat{\mathcal{B}}_{\text{new}} : \|b' - b''\|_1 \geq \delta_{\text{single}}$  then
38          $\hat{\mathcal{B}}_{\text{new}} \leftarrow \hat{\mathcal{B}}_{\text{new}} \cup \{b'\}$ 
39          $w_{\text{new}} \leftarrow w_{\text{new}} \cup \{w'\}$ 
40       else
41          $k \leftarrow \text{argmin}_{b'' \in \hat{\mathcal{B}}_{\text{new}}} \|b' - b''\|_1$ 
42          $w_{\text{new}}[k] \leftarrow w_{\text{new}}[k] + w'$ 
43   return  $\hat{\mathcal{B}}_{\text{new}}, w_{\text{new}}$ 

```

All experiments were implemented and executed using Julia [12] with the POMDPs.jl framework [18]. Problem implementations were based primarily on originating papers, with additional references to the Multiagent Systems Planning Page [39] and the Dec-POMDP page [2] to ensure consistency with previous work. For context, we include the best-reported results from Dec-POMDP solvers when available, noting that our approach's use of communication makes these comparisons informative but not equivalent.

6.1 Benchmark Problems

We tested MCAS on several Dec-POMDP benchmarks: Decentralized Tiger [31], Broadcast Channel [23], Meeting in a 2×2 Grid [11], Meeting in a 3×3 Grid [4], Cooperative Box Pushing [37], Wireless Networking [34], and Mars Rover [5]. For detailed problem descriptions and implementations, we refer readers to the original papers and our accompanying repository.

The original problems were designed without considering communication. In our experiments, we found that when we allowed one agent, using only its individual observations, to control all agents, it often achieved performance similar to a full MPOMDP (with shared observations and actions). To better demonstrate the value of integrating different beliefs, we introduced modifications to increase difficulty and emphasize the importance of different agent observations. For instance, in the original Meeting in 2×2 Grid problem, agents started with known locations and faced no penalties for wall collisions, enabling simple but effective policies like always moving towards a corner.

We use qualifiers to denote problem modifications from the original implementation in our results:

- *UI*: Changed the initial belief to a uniform distribution.
- *WP*: Added penalties to make action selection more consequential (e.g., penalties for wall collisions or message sending).
- *DP*: Modified Broadcast problem probabilities for a three-agent scenario (buffer fill probabilities of 0.2, 0.4, 0.4 for agents one, two, and three, respectively).
- *SS*: For Meet 2×2 , changed starting positions from corners to same row or column.
- *AG*: In Meet 3×3 , rewarded agents for meeting at any grid location, not just two corners.
- *SO*: Introduced stochastic observations in Box Push (50% chance of correct observation, 50% of no observation).
- *5G*: Added an additional sampling site to Mars Rover, accessible from the original top-right location.
- *Meet 19*: Expanded version of Meet 2×2 with 27 grid locations ($|S|^n$ with n agents). Observation space expanded to include *no walls* and *both walls* in addition to the original left and right wall observations.

6.2 Baseline Methods and Implementation Details

We compared MCAS against the following baselines:

- *MMDP*: Multiagent MDP assuming full observability.
- *MPOMDP*: Multiagent POMDP with centralized control.
- *MPOMDP-C*: MPOMDP policy with beliefs generated by conflating the true individual agent beliefs.
- *MCAS- α* : MCAS using alpha vector indices instead of actions, providing more refined subspaces for pruning. Used conflation with similarity parameters δ_{single} and δ_{joint} set to 10^{-5} .
- *MCAS*: As described in section 5.4, using same parameters as *MCAS- α* with maximum estimated beliefs $\bar{B}_{\text{max}} = 200$.
- *MPOMDP-I*: Single agent controls all, using only its individual observations.

Table 1: Average cumulative discounted reward (with 95 % confidence intervals) for various Dec-POMDP problems.

Problem	Qualifiers	# Agents	Solution Method							
			MMDP	MPOMDP	MPOMDP-C	MCAS- α	MCAS	MPOMDP-I	Dec-POMDP	Independent
Dec-Tiger	—	2	200.0	59.5 \pm 0.9	59.5 \pm 0.9	58.5 \pm 0.9	58.5 \pm 0.8	34.3 \pm 1.7	13.5 [35]	-68.1 \pm 3.5
	—	3	300.0	108.5 \pm 1.0	108.5 \pm 1.0	108.5 \pm 1.0	108.5 \pm 1.0	82.1 \pm 1.5	—	-95.5 \pm 4.1
	—	4	400.0	153.0 \pm 0.7	153.0 \pm 0.7	152.8 \pm 0.7	152.8 \pm 0.7	121.3 \pm 1.5	—	-121.4 \pm 4.4
Broadcast	—	2	9.4	9.4 \pm 0.0	9.4 \pm 0.0	9.4 \pm 0.0	9.4 \pm 0.0	9.4 \pm 0.0	9.3 [29]	7.6 \pm 0.1
	DP, WP	3	6.7	6.6 \pm 0.0	6.6 \pm 0.0	6.6 \pm 0.0	6.6 \pm 0.0	5.5 \pm 0.0	—	-0.6 \pm 0.1
Meet 2 \times 2	—	2	8.0	6.4 \pm 0.1	6.1 \pm 0.2	6.1 \pm 0.2	6.1 \pm 0.2	5.9 \pm 0.1	6.1 [3] [†]	1.7 \pm 0.1
	SS	2	8.4	6.9 \pm 0.1	6.8 \pm 0.1	6.8 \pm 0.1	6.8 \pm 0.1	6.8 \pm 0.1	7.0 [29] [†]	2.3 \pm 0.1
	UI, WP	2	8.7	5.8 \pm 0.2	5.3 \pm 0.2	5.3 \pm 0.2	5.3 \pm 0.2	4.5 \pm 0.2	—	3.5 \pm 0.2
Meet 3 \times 3	—	2	5.9	5.8 \pm 0.1	5.8 \pm 0.1	5.8 \pm 0.1	5.7 \pm 0.1	3.6 \pm 0.1	5.8 [16]	3.7 \pm 0.1
	AG, UI, WP	2	8.1	7.3 \pm 0.1	7.3 \pm 0.1	7.3 \pm 0.1	7.1 \pm 0.1	3.5 \pm 0.1	—	2.8 \pm 0.1
	AG, UI, WP	3	7.2	6.4 \pm 0.1	6.4 \pm 0.1	6.4 \pm 0.1	6.2 \pm 0.1	1.0 \pm 0.1	—	1.7 \pm 0.1
Meet 19	UI, WP	2	6.3	2.2 \pm 0.1	2.1 \pm 0.1	2.0 \pm 0.1	1.6 \pm 0.1	0.6 \pm 0.1	—	0.6 \pm 0.1
Box Push	—	2	240.1	222.9 \pm 2.2	223.4 \pm 2.1	223.4 \pm 2.1	223.0 \pm 2.2	199.6 \pm 2.6	224.4 [16]	163.6 \pm 3.4
	SO	2	240.1	204.3 \pm 2.5	203.4 \pm 2.5	203.2 \pm 2.5	199.8 \pm 2.5	178.8 \pm 2.7	—	138.5 \pm 3.8
Wireless	—	2	-143.6	-152.8 \pm 2.3	-152.8 \pm 2.3	-152.8 \pm 2.3	-153.0 \pm 2.4	-152.8 \pm 2.3	-167.1 [29] [‡]	-219.8 \pm 3.9
	WP	2	-154.5	-165.8 \pm 2.4	-166.5 \pm 2.4	-166.5 \pm 2.4	-166.5 \pm 2.4	-172.4 \pm 2.3	—	-240.2 \pm 4.1
Mars Rover	—	2	29.2	29.0 \pm 0.1	29.0 \pm 0.1	29.0 \pm 0.1	29.0 \pm 0.1	24.4 \pm 0.3	26.9 [16]	26.0 \pm 0.2
	UI	2	24.9	23.9 \pm 0.1	23.9 \pm 0.1	23.9 \pm 0.1	19.8 \pm 0.2	16.4 \pm 0.2	—	15.3 \pm 0.2
	UI	3	26.2	25.2 \pm 0.1	25.2 \pm 0.1	25.2 \pm 0.1	23.8 \pm 0.2	19.7 \pm 0.1	—	16.6 \pm 0.1
	5G, UI	2	21.4	20.7 \pm 0.1	20.7 \pm 0.1	20.7 \pm 0.8	18.0 \pm 0.2	14.8 \pm 0.1	—	13.1 \pm 0.2

^{*} The papers reporting the best scores for Meeting 2 \times 2 do not discuss the initial state. We associated the best-reported result with an initial condition based on the MPOMDP solutions (which is an upper bound on Dec-POMDP results). Other reported scores: [35]: 6.9, [5]: 5.6.

[†] Dibangoye et al. [16] reported a value of -140.4, but we were unable to verify the implementations details. The reported value -140.4 is better than the performance of the MPOMDP on our implementation which implies there is a difference in implementation. Previously highest reported score prior to MacDermed and Isbell [29] was -175.4 by Pajarinen and Peltonen [35].

- *Independent*: Agents execute individual policies (assuming control of other agents), ignoring messages.
- *Dec-POMDP*: Best reported results from literature (experiments not conducted by us).

All POMDP policies were computed using SARSOP [28]. Experiments for POMDP-based methods were conducted on a MacBook Pro with an Apple M1 Max processor and 32 GB of memory, running each scenario 2000 times. Results for these methods are reported with 95 % confidence intervals. MMDP results represent the converged policy value and are reported without confidence intervals. Most problems used 50 time steps with a discount factor of 0.9, while the Wireless Network problem used 450 steps and a 0.99 discount factor.

6.3 Results

The results presented in table 1 offer several interesting insights into the performance of our proposed MCAS algorithm across various Dec-POMDP benchmarks. One notable observation is the consistent performance of MPOMDP-C compared to MPOMDP across all problems. This suggests that using conflation to combine beliefs is an effective approach, particularly in these scenarios where observations are independent. The similarity in performance indicates that conflation successfully integrates information from multiple agents without significant loss of decision-making quality.

MCAS- α consistently matches or closely approximates MPOMDP-C results, implying accurate belief estimates and effective use of the refined subspace information provided by alpha vector indices. MCAS (using actions) performs marginally worse than MCAS- α and MPOMDP-C, but still achieves comparable results despite having a less refined belief subspace for pruning. This performance indicates that MCAS can maintain an effective joint belief estimate with a belief subspace defined only by shared actions.

MCAS effectively pruned beliefs, keeping $|\mathcal{B}^{1,j}|$ relatively low. The maximum set size limit (\bar{B}_{\max}) was reached in only two problems: 3.2 % of Meet 19 and 87.8 % of Box Push-SO runs. The largest performance decreases for MCAS compared to MCAS- α occurred in Meet 19, Box Push-SO, Mars Rover-UI, and Mars Rover-5G-UI. This difference is due to MCAS- α 's more effective pruning. Table 2 shows the maximum estimated belief set sizes for problems with a noticeable increase for MCAS. Despite larger set sizes, MCAS still achieved high performance approaching that of MCAS- α . We anticipate this gap will decrease with improved belief selection. The average and maximum set sizes for all problems are provided in table 3.

An interesting pattern emerges when comparing MPOMDP and MPOMDP-I results. In problems like Broadcast, Meeting, and Wireless, these approaches yield similar performance, suggesting limited benefit in maintaining estimates of other agents' beliefs. In such scenarios, even sharing observations provides no advantage over

Table 2: Maximum size of $\hat{\mathcal{B}}^{1,j}$ per simulation.

Problem	Qualifiers	# Agents	Solution Method	
			MCAS- α	MCAS
Meet 3×3	—	2	1.0 ± 0.0	2.5 ± 0.0
Meet 19	UI, WP	2	1.5 ± 0.0	16.8 ± 1.6
Box Push	SO	2	4.8 ± 0.1	192.1 ± 1.2
Wireless	—	2	1.0 ± 0.0	18.0 ± 0.9
Mars Rover	UI	2	1.0 ± 0.0	2.0 ± 0.0
	5G, UI	2	1.0 ± 0.0	3.0 ± 0.0

beliefs using only individual observations. This insight could be valuable for simplifying processes in certain types of multiagent problems, though determining which agent should take the lead in such cases would require further consideration.

A challenge in conducting these experiments was the generation of MPOMDP policies. While this process is substantially more tractable compared to Dec-POMDP solvers, the complexity of solving MPOMDPs still grows exponentially with the number of agents. The online execution of MCAS, on the other hand, did not pose a major computational burden. All simulations were conducted on a standard laptop, demonstrating the algorithm’s efficiency. This balance between the offline computational load of policy generation and the lightweight online execution makes MCAS a promising approach for more practical multiagent problems.

7 CONCLUSIONS AND FUTURE WORK

This paper introduced the Multiagent Control via Action Suggestions (MCAS) algorithm, a new approach to coordinating multiple agents in partially observable environments. By leveraging suggested actions as a form of communication, MCAS demonstrated performance comparable to centralized methods across various Dec-POMDP benchmarks, while maintaining computational efficiency. The algorithm effectively prunes the reachable belief space enabling accurate belief inference of other agents which allows for the estimation of a joint belief and improved decision making.

Though the results of MCAS are promising, there are many opportunities for future research. A key area is a deeper theoretical analysis of MCAS. This analysis includes studying the convergence properties of the belief estimation process, establishing performance bounds relative to centralized methods, and investigating the information-theoretic properties of action-based communication in multiagent settings. Another important area is relaxing the strong assumptions made in this work. For example, investigating scenarios where agents lack access to others’ exact policies could reveal how similar surrogate policies need to be to maintain performance. Exploring cases where agents do not always follow the coordinator’s suggestions would enhance robustness. Extending the ideas of MCAS to online solvers like AdaOPS [41] and BetaZero [30] is also an important area of research for solving larger, more complex problems. This integration would require developing efficient methods to estimate belief subspaces in real-time and handle the stochastic nature of online policies in belief inference.

Our results indicate that action-based communication can be a powerful tool for multiagent coordination, potentially bridging

the gap between decentralized and centralized approaches. As we continue to refine and extend these methods, we move closer to realizing the full potential of collaborative decision making in complex, partially observable environments. Importantly, this approach lays the groundwork for more intuitive coordination in human-agent teams, opening up exciting possibilities for mixed-initiative planning and decision making in real-world applications.

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A ADDITIONAL RESULTS

Table 3: Average and Maximum size of $\hat{\mathcal{B}}^{1,j}$ per simulation (all problems).

Problem	Qualifiers	# Agents	Average $ \hat{\mathcal{B}}^{1,j} $		Max $ \hat{\mathcal{B}}^{1,j} $	
			MCAS- α	MCAS	MCAS- α	MCAS
Dec-Tiger	—	2	1.0 \pm 0.0	1.0 \pm 0.0	1.6 \pm 0.0	1.6 \pm 0.0
	—	3	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0
	—	4	1.0 \pm 0.0	1.0 \pm 0.0	1.1 \pm 0.0	1.1 \pm 0.0
Broadcast	—	2	1.0 \pm 0.0	1.0 \pm 0.0	2.0 \pm 0.0	2.0 \pm 0.0
	DP, WP	3	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0
Meet 2 \times 2	—	2	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0
	SS	2	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0
	UI, WP	2	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0
Meet 3 \times 3	—	2	1.0 \pm 0.0	1.1 \pm 0.0	1.0 \pm 0.0	2.5 \pm 0.0
	AG, UI, WP	2	1.0 \pm 0.0	1.1 \pm 0.0	1.0 \pm 0.0	1.2 \pm 0.0
	AG, UI, WP	3	1.0 \pm 0.0	1.1 \pm 0.0	1.0 \pm 0.0	1.3 \pm 0.0
Meet 19	UI, WP	2	1.0 \pm 0.0	4.9 \pm 0.5	1.5 \pm 0.0	16.8 \pm 1.6
Box Push	—	2	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.5 \pm 0.0
	SO	2	1.8 \pm 0.0	78.0 \pm 1.6	4.8 \pm 0.1	192.1 \pm 1.2
Wireless	—	2	1.0 \pm 0.0	3.5 \pm 0.1	1.0 \pm 0.0	18.0 \pm 0.9
	WP	2	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0
Mars Rover	—	2	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0
	UI	2	1.0 \pm 0.0	1.2 \pm 0.0	1.0 \pm 0.0	2.0 \pm 0.0
	UI	3	1.0 \pm 0.0	1.0 \pm 0.0	1.0 \pm 0.0	1.1 \pm 0.0
	5G, UI	2	1.0 \pm 0.0	1.2 \pm 0.0	1.0 \pm 0.0	3.0 \pm 0.0